More Portfolio Survival Studies

John Norstad

j-norstad@northwestern.edu
http://www.norstad.org

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Abstract

We examine the impact of time, investment expenses, expected return and volatility on the portfolio survival problem.

The risk of outliving your money in retirement increases rapidly with time horizon and does not begin to level off until very long horizons.

Investment expenses are arguably the single most important factor affecting survival rates. Retirees who wish to minimize the risk of outliving their money are well-advised to use low-cost index funds and, if possible, forgo the services of expensive investment advisors.

Portfolio efficiency has a significant impact on survival rates and thus good diversification over a wide variety of asset classes is an important issue.

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1 The Model

We use the random walk model developed in [2]:

$$\begin{aligned}\frac{\Delta s}{s} &= e^{\mu\Delta t+\sigma\Delta X} - 1 + \frac{k\Delta t}{s} \quad \text{where } \Delta X \text{ is } N[0,\Delta t] \\ s(t+\Delta t) &= s(t)e^{\mu\Delta t+\sigma\Delta X} + k\Delta t\end{aligned}$$

t is the time in years.

Δt is a time interval.

$s(t)$ is the portfolio value at time t.

μ is the expected continuously compounded yearly rate of return for the portfolio.

σ is the standard deviation of the continuously compounded yearly returns.

k is a constant yearly amount added to ($k > 0$) or subtracted from ($k < 0$) the portfolio in even installments at the end of each time interval Δt.

We use $k = -5$ for a 5% withdrawal rate and $\Delta t = 1/12 = 0.08333\ldots$ to model monthly withdrawals from the portfolio. We use $s(0) = \$100$ for the initial portfolio value.

Given values for the parameters μ, σ, and k, we use a computer program to run 100,000 Monte Carlo simulations under this model of portfolio growth over t years. The *survival rate* is the percentage of these simulations with ending values $s(t) \geq 0$.

We use a model portfolio of 5% cash, 25% bonds, and 70% stocks. This is a fairly typical moderately aggressive portfolio.

We estimate μ and σ using historical time series data from 1926 through 1994.[1]

$$\begin{aligned} C &= \text{90 day US Treasury bills} \\ B &= \text{20 year US Treasury bonds} \\ S &= \text{S\&P 500 stocks} \\ I &= \text{Consumer price index}\end{aligned}$$

Given investment expenses of e we construct a time series N for the nominal return of the portfolio after expenses:

$$N = 0.05 \times C + 0.25 \times B + 0.70 \times S - e$$

We adjust for inflation by converting to real returns relative to the CPI:

$$R = \frac{N - I}{1 + I}$$

[1] The data is from Table 2-4 in [1].

We convert to continuous compounding and take the mean and standard deviation:

$$\begin{aligned}\mu &= \mathrm{E}(\log(1+R)) \\ \sigma &= \mathrm{Stdev}(\log(1+R))\end{aligned}$$

Note that this study is adjusted for inflation. We keep our withdrawal amount $k = -5$ constant and in our parameter estimation we convert from nominal to real rates of return. For our purpose of computing survival rates, this is equivalent to using nominal rates of return and varying k each month to adjust the withdrawal amount for inflation. We omit the trivial proof.

2 Survival as a Function of Time

In the first experiment we use a constant investment expense ratio of $e = 35$ basis points. The parameter estimation gives the following values for μ and σ:

$$\mu = 5.1860\%$$
$$\sigma = 14.7012\%$$

We vary t from 1 year to 100 years in increments of 1 year. Figure 1 shows the result.

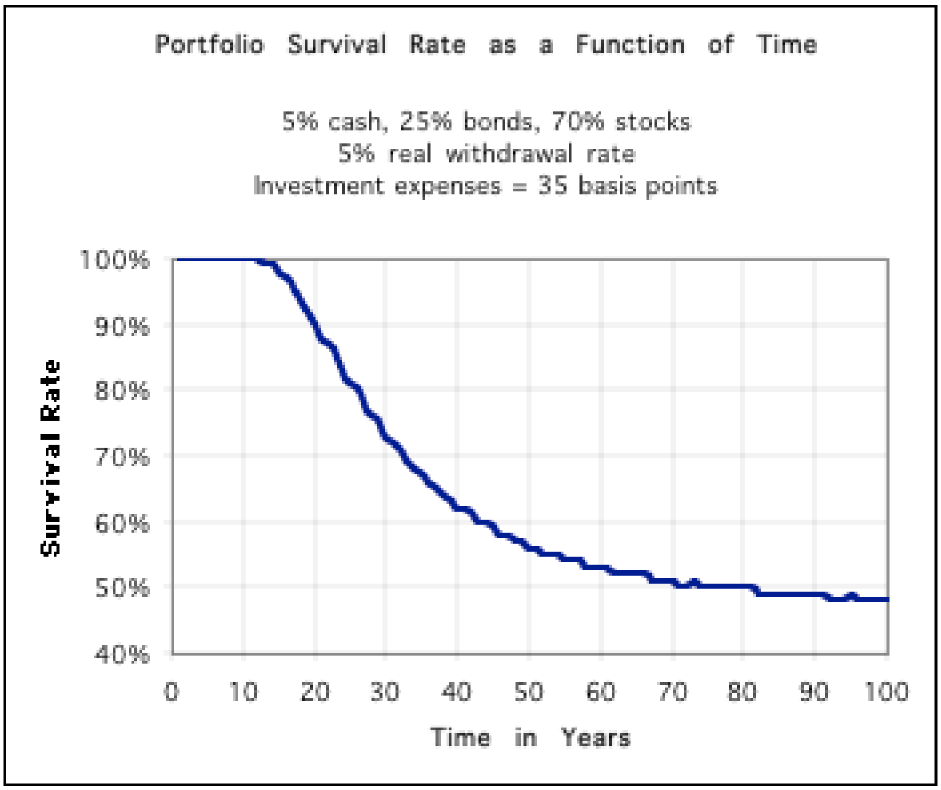

Figure 1: Portfolio Survival as a Function of Time

For the first 12 years the survival rate is 100% (to the nearest whole percentage). After 12 years the survival rate begins to drop rapidly. The survival rates are 90% at 20 years, 73% at 30 years, and 62% at 40 years. At very long time horizons the survival rate begins to level out and decrease much more slowly.

3 Survival as a Function of Investment Expenses

In this experiment we fix the time period t at 30 years and vary the investment expense ratio from 20 basis points to 200 basis points in increments of 5 basis points. Figure 2 shows the result.

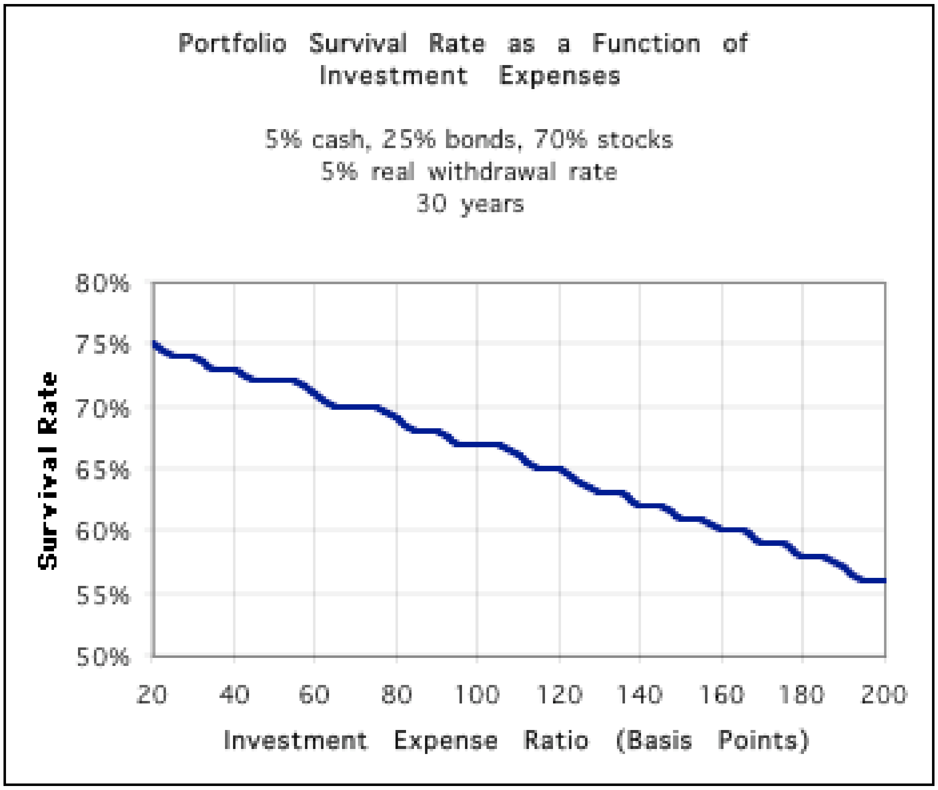

Figure 2: Portfolio Survival as a Function of Expenses

At 20 basis points the survival rate is 75%. As the expense ratio increases the survival rate drops rapidly, to only 56% at 200 basis points.

Investment expenses are clearly a critical factor in portfolio survival. 20 basis points is about the minimum one can achieve in today's mutual fund industry, by using index funds offered by very low-cost companies like Vanguard.

Note that when estimating investment expenses one must consider not only the expense ratio of the funds but also the impact of adviser fees (if any) and portfolio turnover. Common estimates of the average total expenses for actively managed mutual funds without any adviser fees are in the neighborhood of 200 basis points or even higher. Adviser fees typically add another 100-200 basis points to expenses.

There is a clear lesson to be learned from this experiment. Retirees who are funding their retirement via withdrawals from an investment portfolio should make every attempt possible to minimize expenses. This is a very strong argument both in favor of index funds and in favor of managing your own portfolio without the assistance of an expensive advisor.

4 Survival as a Function of Expected Return

Our model portfolio of 5% cash, 25% bonds and 70% stocks with low investment expenses of 35 basis points has the following expected continuously compounded real rate of return and standard deviation of return:

$$\mu = 5.1860\%$$
$$\sigma = 14.7012\%$$

In our third experiment we see what happens if we hold the volatility parameter fixed and vary the expected return parameter μ in a neighborhood surrounding the 5.1860% value of our model portfolio. We vary μ from 4.2% to 6.2% in increments of 0.1%. We keep the time parameter t fixed at 30 years.

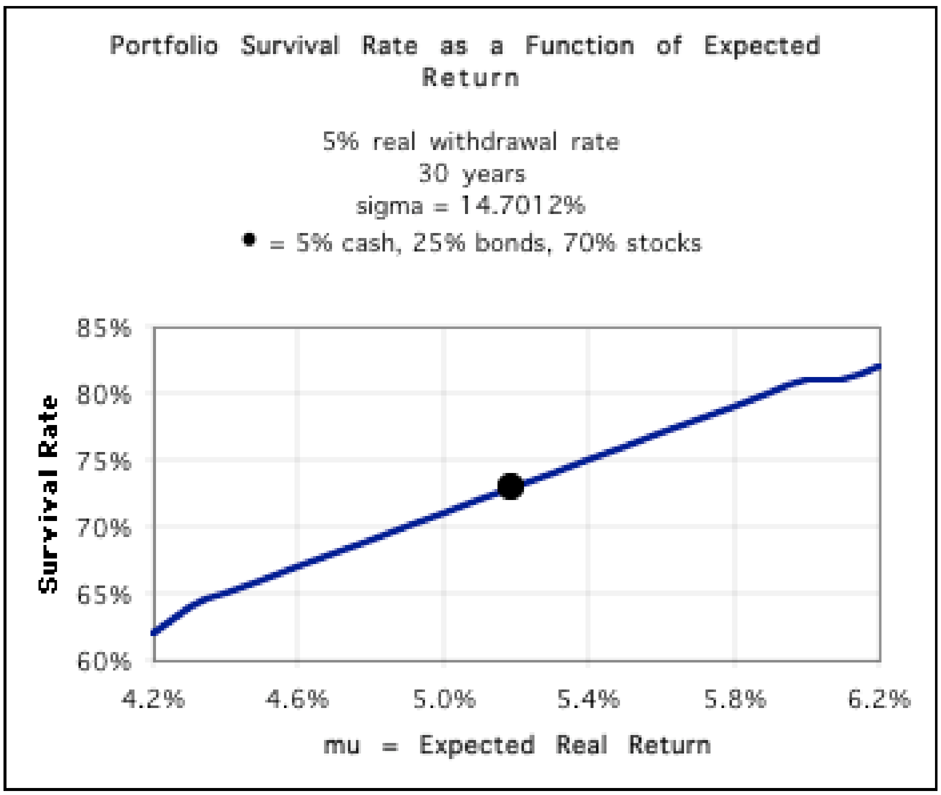

Figure 3: Portfolio Survival as a Function of Expected Return

This experiment shows that if we can increase expected return without increasing volatility we significantly increase the probability that our portfolio will survive for 30 years. A 1% increase in expected return improves the survival rate from 73% to 82%. This is a major improvement.

5 Survival as a Function of Volatility

Our last experiment is a partner of the expected return experiment. We once again fix investment expenses at 35 basis points and the time horizon at 30 years. This time we hold the expected return parameter μ fixed and vary the volatility parameter σ in the neighborhood of the value 14.7012% of our model portfolio. We vary σ from 12.7% to 16.7% in increments of 0.2%.

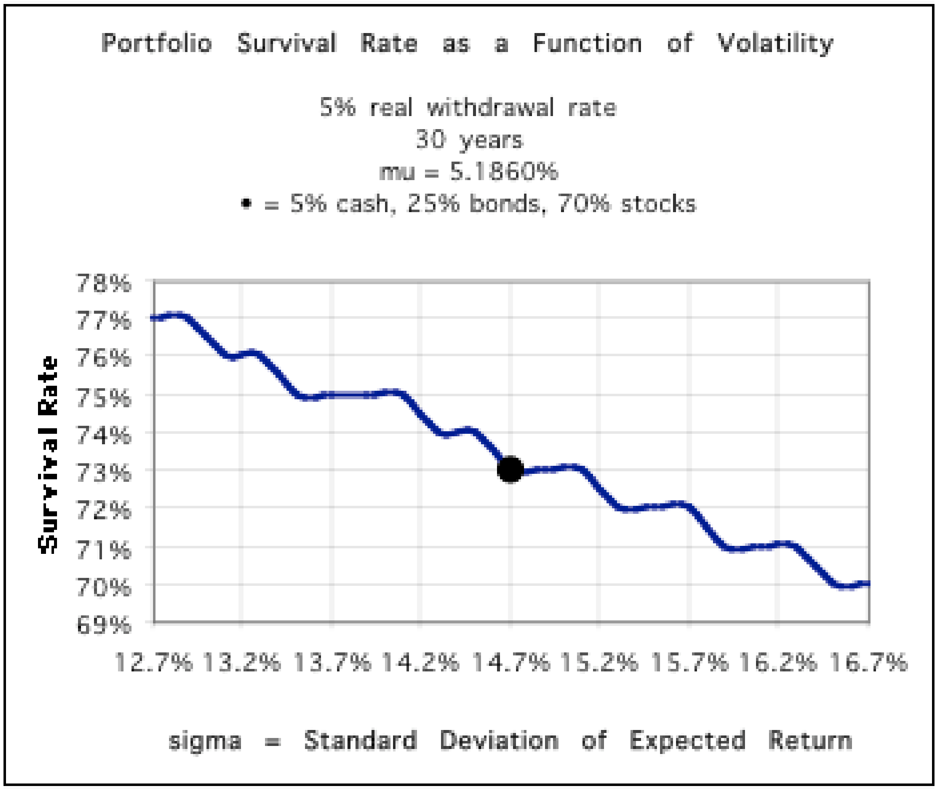

Figure 4: Portfolio Survival as a Function of Volatility

This experiment shows that if we can decrease volatility without decreasing expected return we increase the survival rate. For example, a 2% decrease in volatility improves the survival rate from 73% to 77%.

6 The Benefits of Diversification

We constructed our model portfolio using only three simple asset classes: U.S. Treasury bills, 20 year U.S. Treasury bonds, and S&P 500 stocks. Portfolio theory teaches us that we should be able to increase the "efficiency" of our portfolio by diversifying over a larger number of asset classes. Common additional asset classes used for portfolio diversification include small stocks, foreign stocks and bonds, emerging market stocks and bonds, corporate bonds, real estate, and commodities.

Making a portfolio more efficient through diversification increases its expected return, decreases its volatility, or both.

Experiments 3 and 4 show the impact of this increased efficiency on the problem of portfolio survival. What we've seen is that in the neighborhood of our model portfolio a 1% increase in expected return increases the survival rate by about 9%, and a 2% decrease in volatility increases the survival rate by about 4%.

We conclude that good diversification over a large number of asset classes is an important issue. It is also, unfortunately, a notoriously difficult problem.

References

[1] Zvi Bodie and Robert C. Merton. *Finance*. Prentice-Hall, preliminary edition, 1998.

[2] John Norstad. Financial planning using random walks. http://www.norstad.org/finance, Feb 1999.